Biometrics (CSE 40537/60537)

Lecture 2: Fingerprint recognition

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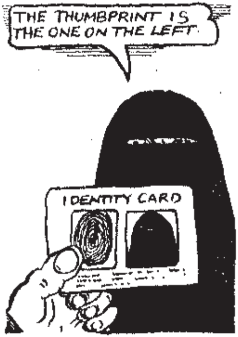


Lecture 2: Fingerprint recognition

Scientific milestones

1. 1684: Nehemian Grew, UK
   - first scientific work on fingerprint structure (description of ridges, valleys and sweat pores)
2. 1686: Marcello Malpighi, University of Bologna
   - first classification of fingerprints (definition of similar patterns possible to be grouped)
   - similar work done later by John Purkinji (1823, Poland) and Sir Edward R. Henry (1899, UK)

Scientific milestones

3. 1880: Henry Faulds, Tokio, Japan
   - first experiments showing high uniqueness of fingerprints
   - first identification of a latent fingerprint left on a bottle (after a theft in Tokio's hospital)
   - skill sharing with Charles Darwin and Francis Galton (Darwin's cousin)
4. 1888: Sir Francis Galton
   - definition and classification of *fingerprint singular points*
   - definition of "minutiae" (a.k.a. *Galton's details*)
   - experiments presenting high temporal stability and uniqueness of fingerprints (estimated identification error probability 1:64 billion)
   - "Fingerprints" book: foundations of the methods used presently in biometric systems

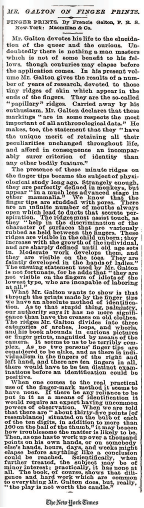
Scientific milestones

Criticism of the "Fingerprints" book, New York Times, 1893

"Carried away by his enthusiasm, Mr. Galton declares that these markings «are in some respects the most important of all anthropological data» (...) What Mr. Galton wants to show is that through the prints made by the finger tips we have an absolute method of identification. As to that stupid thing, palmistry, our authority says it has no more significance than the creases on old clothes."

"When one comes to the real practical use of the finger-mark method it seems to have none. If there be any reliance to be put in it as a means of identification it would require an expert having uncommon powers of observation."

"Scientifically, when further treated, the subject may be of minor interest; practically, it has none at all. The book, of course, shows that diligence and hard work which are common to everything Mr. Galton does, but really «the play is not worth the candle»."



MR. GALTON ON FINGER PRINTS.

FINGER PRINTS. By Francis Galton, F. R. S. New-York: Macmillan & Co.

Mr. Galton devotes his life to the elucidation of the queer and the curious. Undoubtedly there is nothing a man masters which is not of some benefit to his fellows, though centuries may elapse before the application comes. In his present volume Mr. Galton gives the results of a number of years of research, devoted to those tiny ridges of skin which appear in the ends of the fingers. They are the so-called "papillary" ridges. Carried away by his enthusiasm, Mr. Galton declares that these markings "are in some respects the most important of all anthropological data." He makes, too, the statement that they "have the unique merit of retaining all their peculiarities unchanged throughout life, and afford in consequence an incomparably surer criterion of identity than any other bodily feature."

The presence of these minute ridges on the finger tips became the subject of physiological study long ago. Strangely enough, they are perfectly defined in monkeys, but appear "in a much less advanced stage in other mammalia." We know that the finger tips are studded with pores. There are an infinite number of mouths always open which lead to ducts that secrete perspiration. The ridges must assist touch, as they "help in the discrimination of the character of surfaces that are variously rubbed as held between the fingers. These ridges are visible in the child unborn; they increase with the growth of the individual, and are sharply defined until old age sets in. Moderate work develops them, and they are visible on the toes. They are faintly developed in the hands of ladies." The ensuing statement used by Mr. Galton is not fortunate, for he adds that "they are not visible on the fingers of idiots of the lowest type, who are incapable of laboring at all."

What Mr. Galton wants to show is that through the prints made by the finger tips we have an absolute method of identification. As to that stupid thing, palmistry, our authority says it has no more significance than have the creases on old clothes. The ridges Mr. Galton divides into three categories of arches, loops, and whorls, and his book abounds in curious pictures or finger prints, magnified by means of the camera. It seems to us to be terribly complex. As no two persons' finger tips are considered to be alike, and as there is individualism in the fingers of the right and left hand, and there are ten fingers in all, there would have to be ten distinct examinations before an identification could be positive.

When one comes to the real practical use of the finger-mark method it seems to have none. If there be any reliance to be put in it as a means of identification it would require an expert having uncommon powers of observation. When we are told that there are "about thirty-five points [of resemblance] situated on the bulb of each of the ten digits, in addition to more than 100 on the ball of the thumb," it may be seen how troublesome the matter is likely to be. Then, as one has to work up over a thousand points on his own hands, or on somebody else's hands, hours, days, and weeks might elapse before anything like a conclusion could be reached. Scientifically, when further treated, the subject may be of minor interest; practically, it has none at all. The book, of course, shows that diligence and hard work which are common to everything Mr. Galton does, but, really, "the play is not worth the candle."

The New York Times

Bertillonage system ...

http://www.nlm.nih.gov

5. 1880: Alphonse Bertillon, France
   - *Bertillonage* system: lengths of bones + "soft biometrics": height and weight, iris color (not the iris pattern used currently)

... and its collapse in 1903

William West and Will West institutionalized at the same time to the same penitentiary in Leavenworth, Kansas, USA. This significantly undermined the *Bertillonage* system reliability.

Source: "History of Fingerprints", INTERPOL, 2009

Application milestones

1. 1892: Juan Vucetich, Argentina
   - first criminal case, in which the latent fingerprints were used to proof a crime
2. 1910: Edmond Locard, France
   - experiments delivering the number (=12) of matching features required in a court (i.e. defendant is pronounced guilty if at least 12 features match in the sample and reference material)
   - *Exchange Principle*: when committing a crime we always leave something at the crime scene, and take something with us
3. 1903–1924: Common acceptance of fingerprints as an authentication method
   - 1903: Scotland Yard starts to use fingerprints
   - 1915: foundation of the *International Association for Identification*
   - 1924: Fingerprint Identification Division becomes part of *Bureau of Investigation*

IAI logo

What we knew at the beginning of XX century?

1. Individual features of the fingerprints are unique in different fingers and stable in time (**foundation of fingerprint recognition**)
2. Fingerprint ridges and valleys can be grouped by similar shapes observed across the human population (**foundation of fingerprint classification**)

Zooming in the fingerprint surface

1. Base elements: ridges and valleys
2. Level 1 features: core and singular points
3. Level 2 features: minutiae (Galton's details)
4. Level 3 features: sweat pores, incipient ridges, warts, scars, creases

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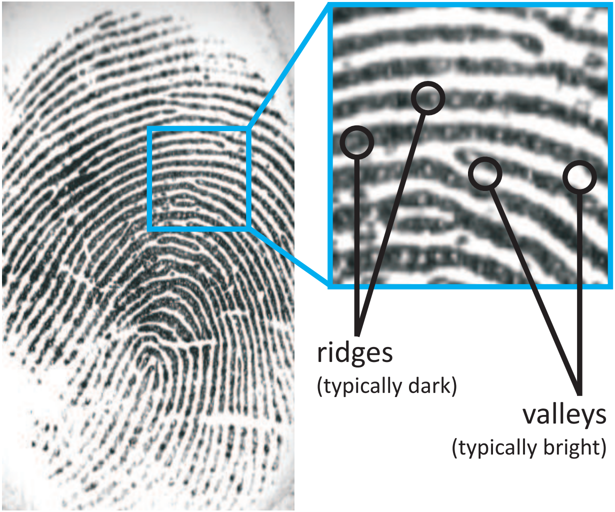
Base elements: ridges and valleys



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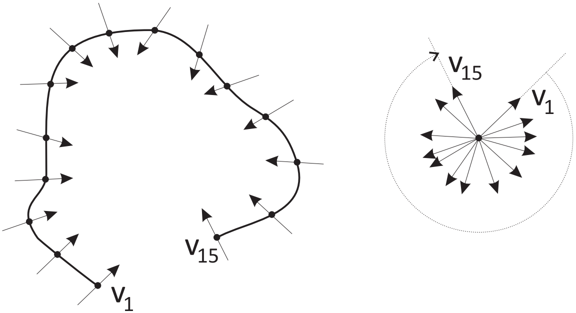
Level 1 features: core and singular points

synthetic fingerprints generated by SFINGE

1. Singular points
   - whorl, loop, delta
   - we need singular points to classify the fingerprints
2. Core
   - the uppermost singular point, or the point where the ridge curvature is the highest (if there is no singular point)
   - we need the core to align fingerprint images (before feature extraction

Detection of singular points

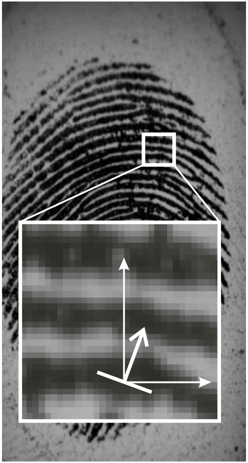
Use of Poincaré index (Kawagoe and Tojo, 1984)



1. In general: total rotation along the curve on a gradient field
2. In fingerprints: we use directional image to estimate ridge directions and calculate the Poincaré index

Detection of singular points

How to calculate directional image?



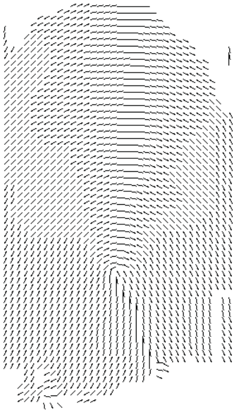
1. Calculate a (discrete) gradient field

$$\nabla(x_i, y_i) = [\nabla_x(x_i, y_i), \nabla_y(x_i, y_i)]^T$$

2. Replace gradients with orthogonal vectors and forget about their orientation $\Rightarrow$ directional field
3. Average directions of the directional field in some small neighborhood, e.g. 8×8 pixels $\Rightarrow$ directional image

Detection of singular points

How to calculate directional image?



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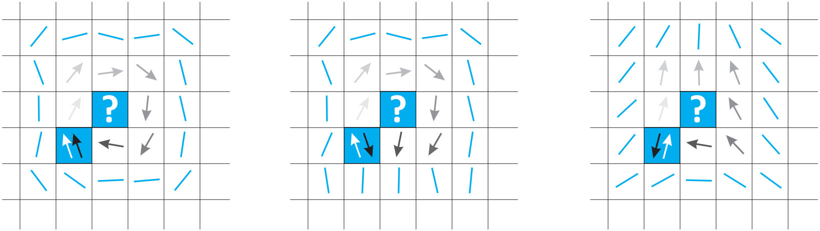
Detection of singular points

How to calculate the Poincaré index?

1. Select random orientation for one, example element of the directional image
2. For the remaining elements select the closest orientation to the neighboring element

Detection of singular points

How to calculate the Poincaré index?



p = 360° (whorl) p = 180° (loop) p = -180° (delta)

3. Travel along the closed curve and sum up the rotations

0^o – no singular points in this location
360^o – we found the whorl
180^o – we found the loop
-180^o – we found the delta

Detection of singular points

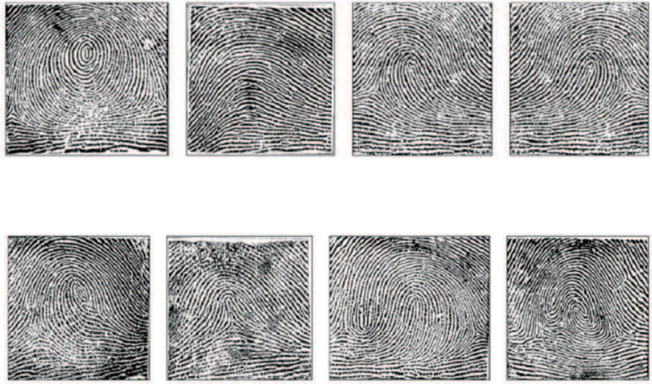
Other popular methods

1. Local properties of ridge pattern: irregularity, curvature, symmetry
   - for instance: ridge irregularity coefficient (Capelli)

$$\rho_{i,j} = 1 - \frac{\| \sum_m \sum_n \mathbf{d}_{m-i,n-j} \|}{\sum_m \sum_n \|\mathbf{d}_{m-i,n-j}\|}$$

2. Image partitioning (use of quantized ridge orientation)

Henry's classification



Source: N. Yager, A. Amin, *Fingerprint verification based on minutiae features*. Pattern Anal. Applic. vol. 7, pp. 94-113, 2004

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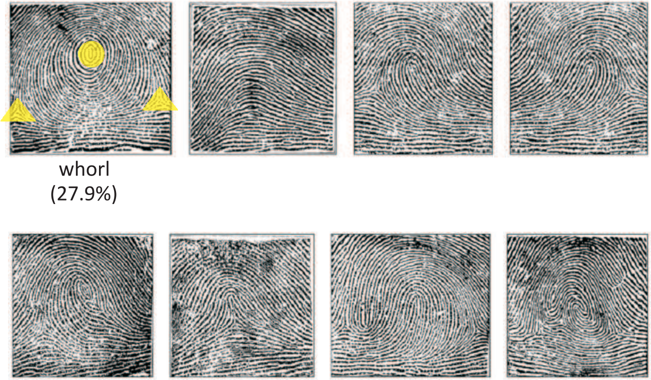

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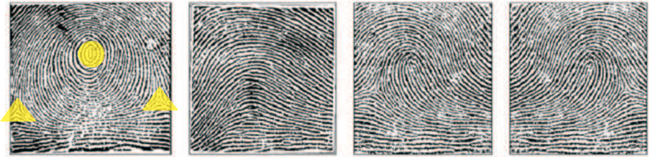
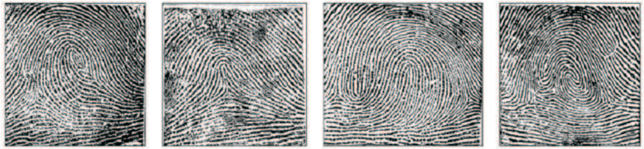


whorl (27.9%)

plain arch (2.7%)



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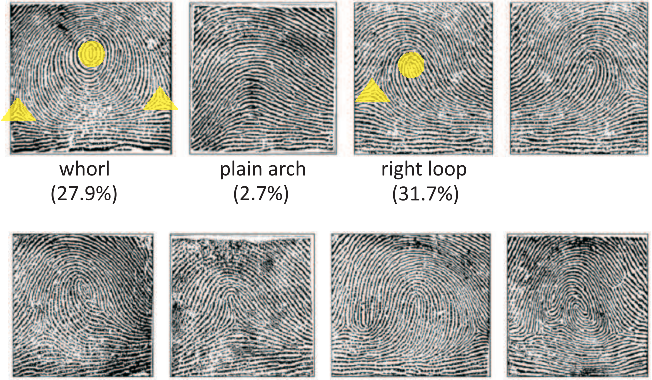

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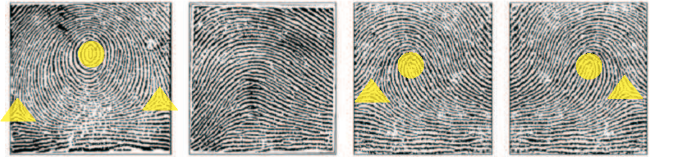
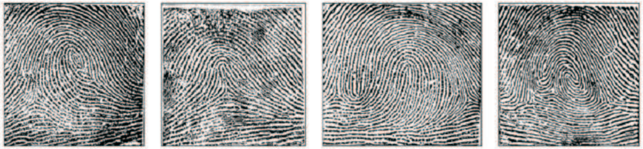


whorl (27.9%) | plain arch (2.7%) | right loop (31.7%) | left loop (33.8%)



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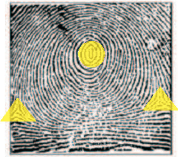
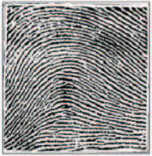
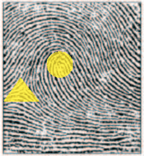


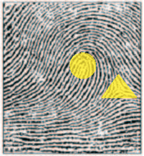
whorl
(27.9%)



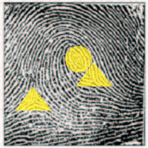
plain arch
(2.7%)



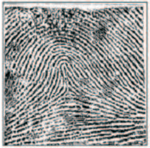
right loop
(31.7%)

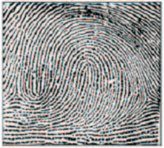


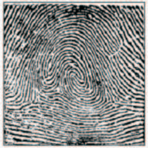
left loop
(33.8%)



central pocket







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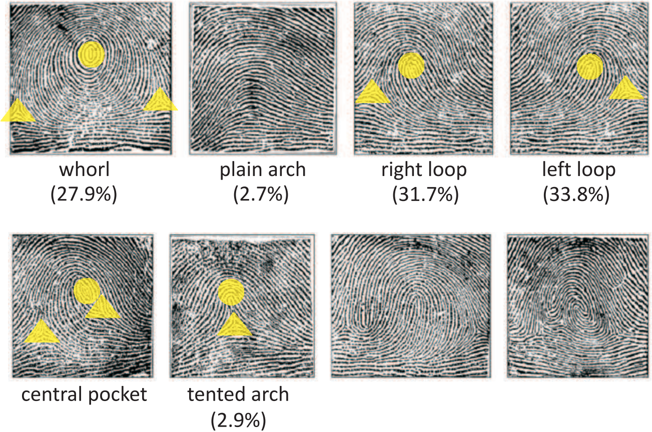

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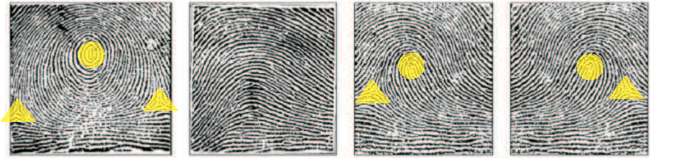

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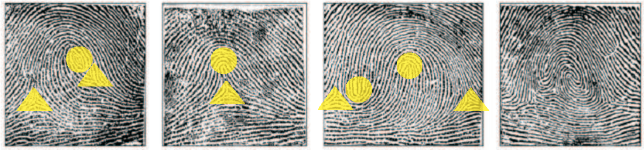
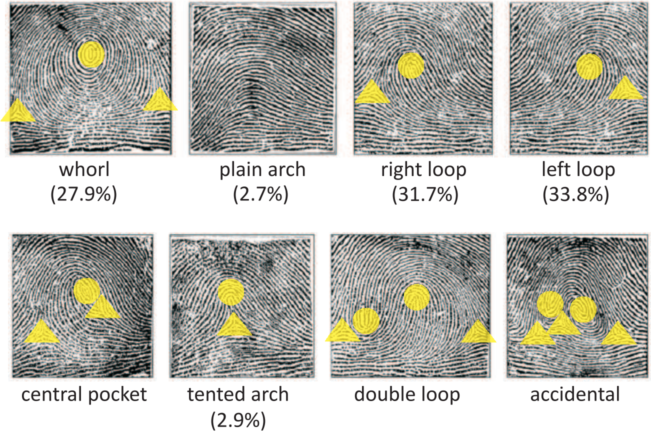


central pocket | tented arch (2.9%) | double loop

Source: N. Yager, A. Amin, *Fingerprint verification based on minutiae features.* Pattern Anal. Applic. vol. 7, pp. 94-113, 2004

Henry’s classification



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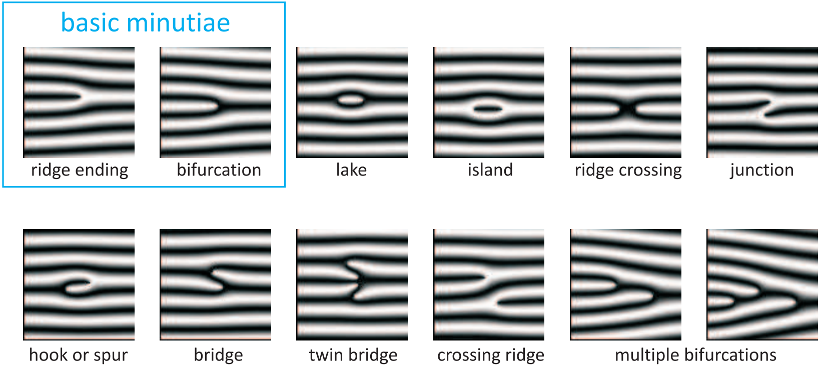
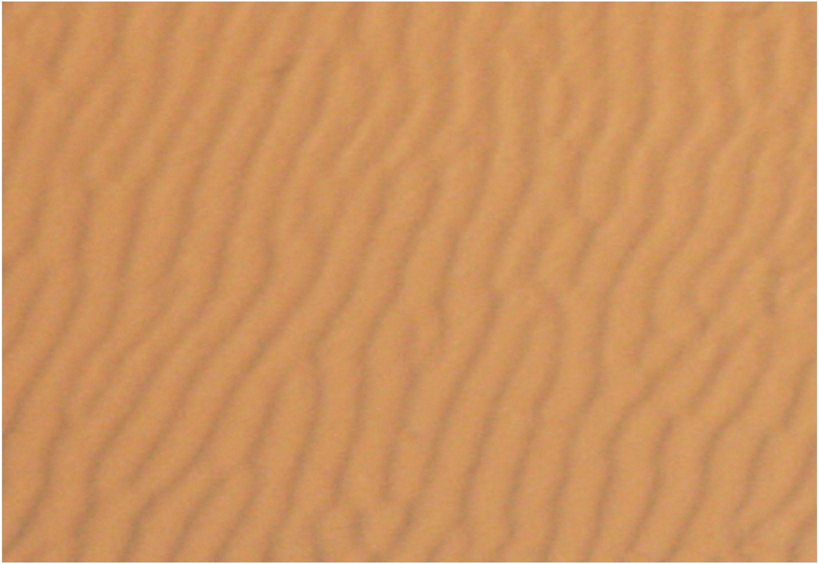
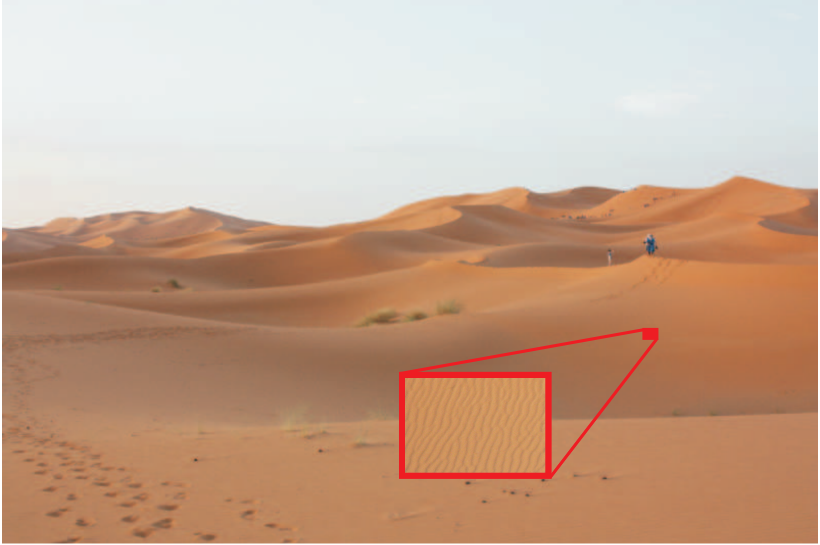
Level 2 features: minutiae (Galton's details)



images found at `www.optel.com.pl`





Erg Chebbi desert, Morocco

Source: freewallpaperspot.com

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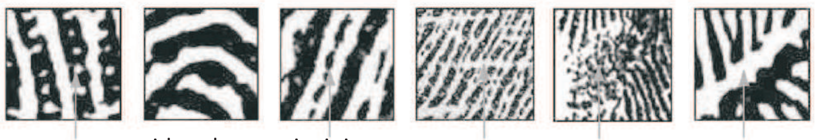
Level 3 features

based on: A. K. Jain, Y. Chen, and M. Demirkus, *Pores and Ridges: High-Resolution Fingerprint Matching Using Level 3 Features,* PAMI, IEEE, 2007



sweat pores (60-250 μm) | ridge shape | incipient ridges | creases (white lines) | warts | scars

1. Recommended image resolution: 1000 dpi
2. Individual features: shape and location of sweat pores, incipient ridges, warts, scars, creases, wrinkles, ridge dots, etc.
3. Helpful when only fragmentary fingerprints are available
4. Popular in fingerprint liveness detection

AFIS vs. biometrics

(AFIS: Automatic Fingerprint Identification System)

1. Criminology
   - Huge data sets: current FBI data set gathers more that 200 million dactyloscopy cards (each card contains fingerprints of all 10 fingers)
   - Different quality of samples (latent and fragmentary fingerprints)
   - Identification times: up to an hour

     Example: a single person would need 67 years (!) to search a database gathering 1.7 million dactyloscopy cards; an AFIS system solves this task in ... 20 minutes

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     Example: a single person would need 67 years (!) to search a database gathering 1.7 million dactyloscopy cards; an AFIS system solves this task in ... 20 minutes
2. Biometrics
   - Similar quality of samples
   - Identification times: fraction of a second

Lecture 2: Fingerprint recognition

- Historical background
- What we observe in fingerprint biometrics?
- **Fingerprint image capture**
- Fingerprint image pre-processing
- Fingerprint recognition methods
- Minutiae detection
- Minutiae matching

Image capture methods

1. Off-line techniques
   - Digitizing (scanning) of dactyloscopy cards
   - Digitizing (photographing) of latent fingerprints
2. On-line techniques
   - Capacitive sensors
   - Optical sensors
   - Piezoelectric (pressure) sensors
   - Thermal sensors
   - Ultrasound sensors
   - Touchless sensors

Digitizing of latent fingerprints

Used by criminology experts, not in biometric systems

Magnetic powder
(fluorescent)

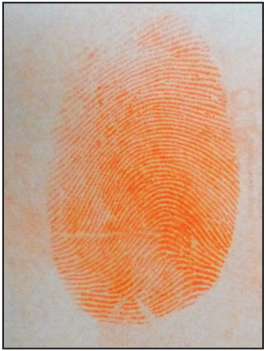
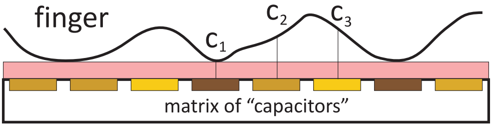


Image in visible light

Image in UV light
(orange filter applied)

Capacitive sensors



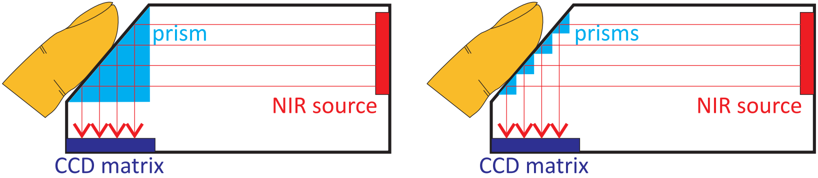
1. Matrix of capacitors: charge depends on the distance between the fingerprint ridge and the sensor; charge interpreted by an AD converter as 8-bit numbers
2. Low cost, but sensitive to noise (dirt, finger moistness)
3. Accuracy decreases if not cleaned (contact with sweat and dirt)
4. Typical resolution: 300 dpi

Capacitive sensors

Fingerprint image sample

Source:
FVC 2002

Optical sensors



1. Total inside reflection (TIR) at the medium boundary (TIR can be observed for angles larger than some critical angle with respect to the normal to the surface)
2. Contact of ridges with the prism ⇒ no TIR in these locations (hence dark regions for ridges)
3. No contact of valleys with the prism ⇒ TIR occurs in these locations (hence bright regions for valleys)
4. Typical resolution: 400–600 dpi

Optical sensors

Fingerprint image samples

"Flat" impression
(sensor: Biometrika FX2000)

"Rolled" impression (from nail to nail)
(sensor: CrossMatch LS320)

Optical sensors

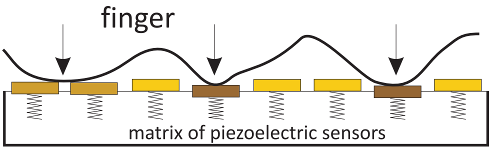
Fingerprint image samples

Thumbs prints

Little, ring, middle and index fingerprints

captured in "4-4-2" mode (sensor: L1 TP4100)

Piezoelectric (pressure) sensors



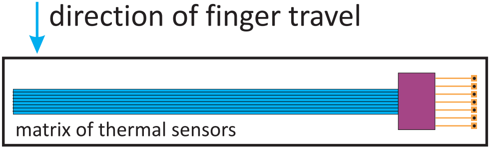
1. Matrix of piezoelectric sensors: pressure interpreted by an AD converter as 8-bit numbers
2. Not sensitive to finger moistness (images of wet and dry fingers are almost identical), and total pressure
3. Typical resolution: 400 dpi

Piezoelectric (pressure) sensors

Fingerprint image sample

Thermal sensors

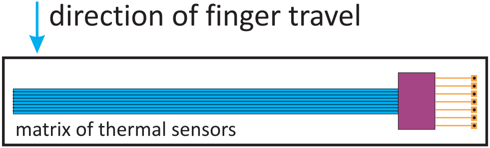
(example: Atmel AT77C101B FingerChip)



1. Measurement of finger swept over the sensor (8×280)
2. AD converter compares temperatures in moments t and $t + \Delta$
3. Accuracy depends on stability of Δ

Thermal sensors

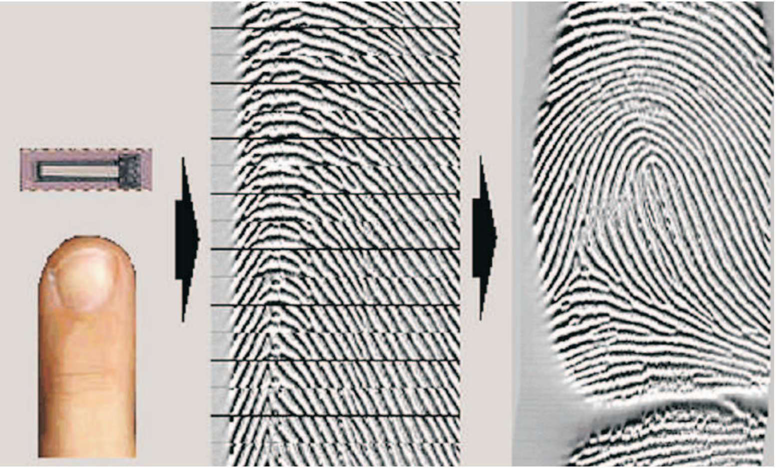
(example: Atmel AT77C101B FingerChip)



4. Small sensor (50μm) induces high resolution (500 dpi)
5. Inverse proportion between sweep speed and accuracy (for typical speed of finger sweep we need more than 500 measurements per second; typical measurement frequency is 2 kHz)

Thermal sensors

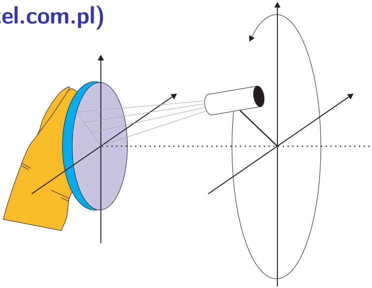
Fingerprint image sample



source: Atmel Fingerchip Witepaper

Ultrasound sensors

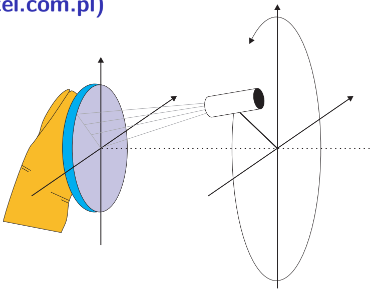
(example: Optel, www.optel.com.pl)



1. Scattering of sound waves on the medium boundary (sensor – finger)
2. Reading the scattered waves by a transducer moving along a circular trajectory, whose axis is perpendicular to the contact surface (to collect scattered signal at different angles, typically 256 directions)

Ultrasound sensors

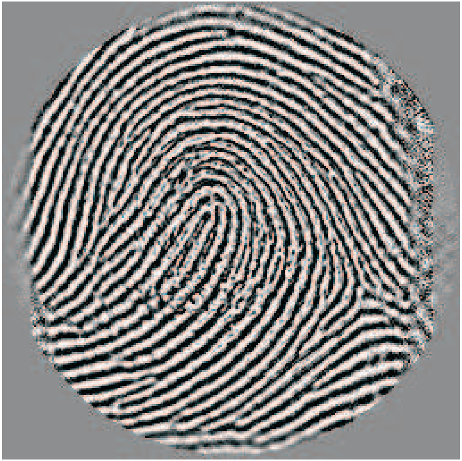
(example: Optel, www.optel.com.pl)



3. Typical resolution: 250 dpi, once all impulse response are colected, the image is reconstructed in single milliseconds
4. May be costly (when compared to capacitive or optical sensors), but difficult to be faked (ultrasound waves can analyse structures underneath the skin)

Ultrasound sensors

Fingerprint image sample



source:
www.optel.com.pl

Commercial fingerprint sensors (examples)

Optical
(Identix)

Capacitive
(Precise Biometrics and Covetek)

Piezoelectric
(BMF/Hitachi)

Thermal
(BioOK and Atmel Fingerchip)

Ultrasound
(Optel)

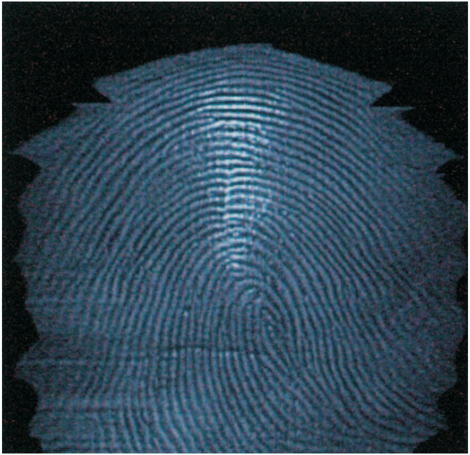
Touchless sensors

SAGEM
Biometrics 2009, London, UK

1. 3D imaging with CCD sensors
2. Transforming of the 3D fingerprint image to the flat image (compliant to ISO/IEC 19794-4)

Touchless sensors

Fingerprint image sample

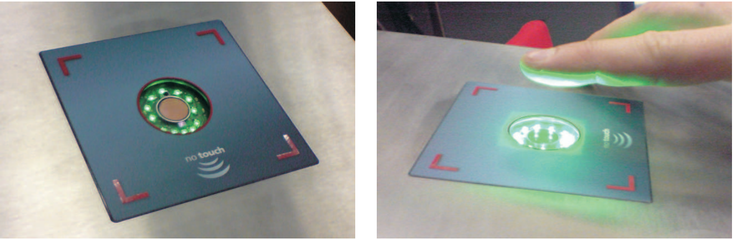


sensor:
MORPHO

Touchless sensors

MORPHO
Biometrics 2010, London, UK

Touchless sensors



TST Biometrics
Biometrics 2010, London, UK

Fingerprint data representation

1. Raw image (not processed)
   ISO/IEC 19794-4:2005
2. Compressed image
   - WSQ (Wavelet Scalar Quantization)
     IAFIS-IC-0110 (V3), WSQ Gray-scale Fingerprint Image Compression Specification 1997
     - calculation of the wavelet transform
     - quantization of the wavelet coefficients
     - Huffman encoding
   - JPEG, JPEG2000, PNG
   - compressed images used in biometric ID and travel documents (due to interoperability requirement)

Fingerprint data representation

3. Use of spectral data representing local image regions
   ISO/IEC 19794-3:2006
   - cosine transform
   - discrete Fourier transform
   - Gabor transform
4. Skeletal image (symbolic representation of ridges)
   ISO/IEC 19794-8:2006
5. Features (e.g. minutiae)
   ISO/IEC 19794-2:2005

Distortions

1. Linear and affine deformations
   - translations and rotations of a finger (severe for small sensors)

     example: finger translation by 2mm (imperceptible for a human) yields 40 pixels of shift in the fingerprint image (for typical 500 dpi sensor)
2. Nonlinear (elastic) deformations
   - different pressure level
   - mapping of 3D elastic object into 2D flat image
   - main source of within-class variability

Distortions

3. Different scanning areas
4. Skin conditions and properties (dry, sweaty, injured)
   - ridges have no physical contact with a sensor
   - incomplete fingerprint images
   - artificial discontinuities and joints of ridges
5. Latent fingerprints left on the sensor
   (impact especially capacitive and optical sensors)
6. Limited accuracy of the sensor

Example: skin condition

Source: D. Maltoni, *et al.* Handbook of Fingerprint Recognition, Second Edition

Normal skin

Dry skin

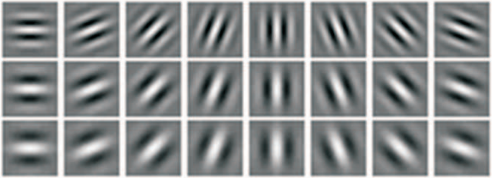
Wet skin

Image enhancement

1. Enhancement of ridges and valleys
   - directional and frequency selectivity: Fourier transform, wavelet transforms (e.g. Gabor)
   - maximum filter response for directions orthogonal to ridges
2. Enhancement of intensity and contrast
   - histogram alignment
   - Laplace filtering

Image enhancement

Example: use of Gabor filtering for enhancement of ridges and valleys



Graphical representation of Gabor filter bank used in fingerprint image enhancement

Original fingerprint image (**left**) and enhanced fingerprint image (**right**)

Source: D. Maltoni, *et al.* Handbook of Fingerprint Recognition, Second Edition

Image segmentation

Image segmentation

1. Analysis of image histograms

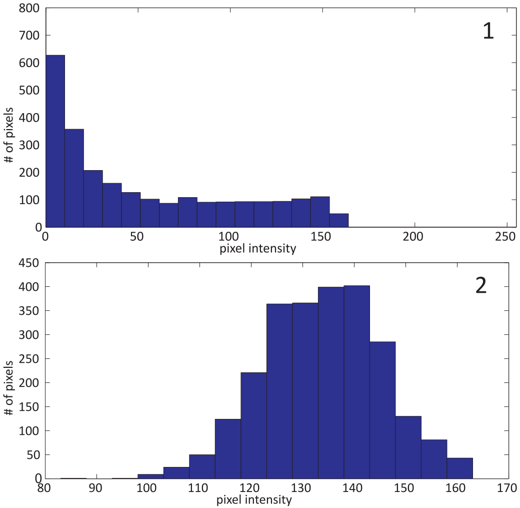


Image segmentation

1. Analysis of image histograms

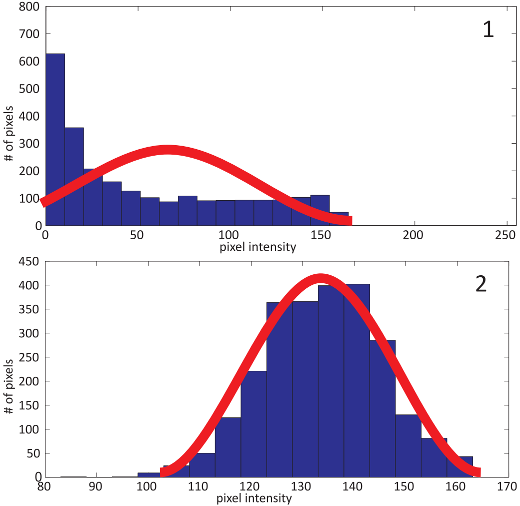
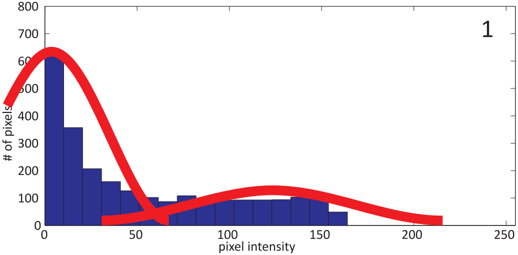


Image segmentation

1. Analysis of image histograms



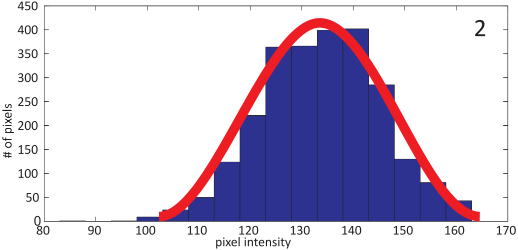


Image segmentation

2. Average variance

In our example:

$$\sigma_1^2 = 2\ 441.12$$

$$\sigma_2^2 = 131.96$$

Modification: variance calculated only in directions orthogonal to ridges

Image segmentation

3. Analysis of image gradient
   - cohesion of gradient directions in ridge areas
   - high values of gradient in ridge areas, low in background

Image segmentation

4. Local Fourier spectrum for image energy estimation
5. Gabor filters (directional and frequency selectivity)
6. Multidimensional metrics (accurate but time-consuming)

Examples:

- variance + intensity + gradient cohesion
- morphological filters + linear classifier

Image segmentation

Example result of fingerprint image segmentation: Gabor filters and a global threshold were use to recognize ‘ridge’ and ’background’ areas.

Classification of fingerprint recognition methods

1. Matching at level 1
    - local properties of directional images
    - image filtering, wavelet transformation + quantization (building a 'fingerprint code')
    - image (local) correlation

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   - fingerprint image representation as a minutiae map (location and rotation, rarely type)
   - minutiae-base methods significantly prevail over all remaining methods

Classification of fingerprint recognition methods

1. Matching at level 1
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2. Matching at level 2
   - fingerprint image representation as a minutiae map (location and rotation, rarely type)
   - minutiae-base methods significantly prevail over all remaining methods
3. Matching at level 3
   - properties of level 3 features: shape, location, orientation (if makes sense), properties in a wavelet domain

Do we need non-minutiae methods?

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1. Higher reliability for low quality fingerprint images
2. Useful for fragmentary fingerprints
3. Additional description of the fingerprints image (higher number of biometric features should help in authentication)

Do we need non-minutiae methods?

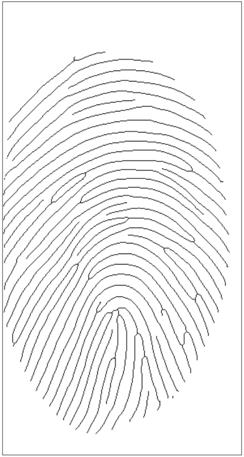
1. Higher reliability for low quality fingerprint images
2. Useful for fragmentary fingerprints
3. Additional description of the fingerprints image (higher number of biometric features should help in authentication)

Answer: YES, sometimes they provide the only authentication possibility when minutiae-based methods fail.

How to locate the minutiae?

Gray scale image

Binary image



Skeletal image

Minutiae detection

Use of gray scale images

1. Ridge tracking (Maio, Maltoni, 1997)
   - Modification 1: adaptive step and binarization threshold (Jiang, Yau, Ser 1999, 2001)
   - Modification 2: simultaneous tracking of a ridge and the neighboring valleys (Liu, Huang, Chan, 2000)
2. Gabor wavelet features + nonlinear classification, e.g. nonlinear neural network (Leung, Engeler, Franck, 1990)
3. Gaussian Mixture Models for local histograms modeling + (Chang, Fan, 2001)
4. Local symmetry of the image (Nilsson, Bigun, 2001)

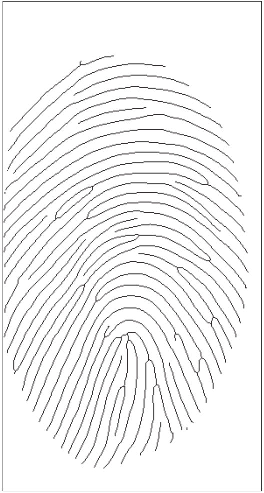
Minutiae detection

Use of binary images

1. Binary image generation
    - use of a global intensity threshold (simple, but low quality)
    - use of a local or adaptive intensity threshold (may be preceded by ridge and valley enhancement)
    - decomposition of a local histogram in directions orthogonal to ridges (Ratha, Chen, Jain, 1995)
2. Ridge tracking (easier than in gray scale image)

Minutiae detection

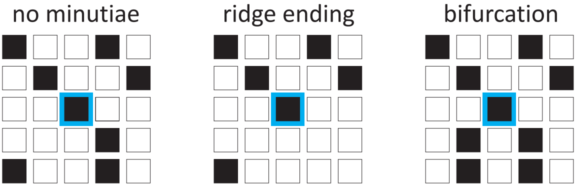
Use of skeletal images



1. Thinning of the binary image
   - solutions inspired by methods used in maps vectorization and handwriting recognition
   - popular solutions are based on mathematical morphology (e.g. multiple erosion)
2. Smoothing of the skeletal image
   - removing very short ridges
   - removing very short joints
   - joining very short ridge breaks

Minutiae detection

Use of skeletal images



based on:
D. Maltoni *et al.*, *Handbook of Fingerprint Recognition*, Springer, 2005

3. Simple analysis of neighboring pixels

number of neighboring 'ridge' pixels provides the answer:

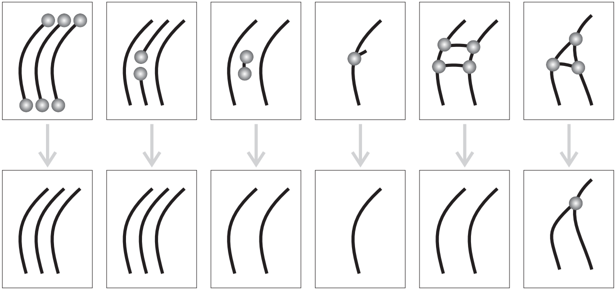
$= 1$: ridge ending

$= 2$: ridge (no minutia)

$\geqslant 3$: bifurcation

Minutiae detection

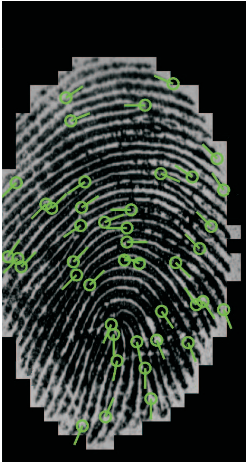
Use of skeletal images

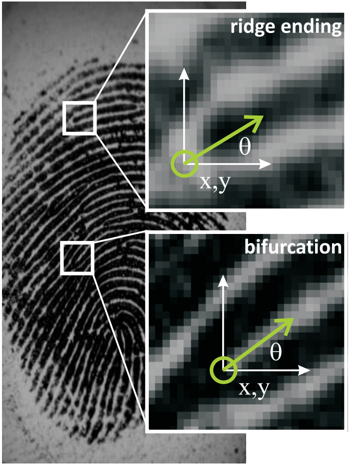


4. Minutiae filtering: removal of 'fake' (excessive) minutiae
   - heuristic methods
     - removing of minutiae located at boundaries
     - removing constellations of minutiae
   - duality of minutiae: simultaneous processing of positive and negative images (Hung, 1993)

Example minutiae map

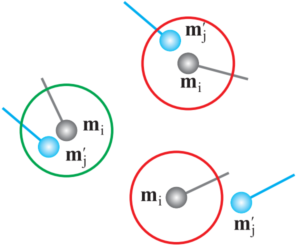
Right: minutiae directions compliant to ISO/IEC FDIS 19794-2 (2011)





Minutiae matching steps

1. Compensation of linear deformations
   - finding a linear mapping between the probe and reference maps of minutiae



2. Compensation of non linear deformations
   - popular and easy: setting the tolerance box: a margin of acceptable difference between properties of two minutiae
   - more accurate, yet time consuming: surface warping (Capelli, Maio, Maltoni, 2001)
   - interesting: use of canonical form of the skeletal images (Senior, Bolle, 2001)

Minutiae matching steps

3. Calculation of the matching score
   - popular: $\frac{\text{number of minutiae that match}}{\text{average number of minutiae in both maps}}$
   - modifications: a) in denominator: number of elements in the common set of minutiae maps, b) minutiae weighting depending on their quality)

Minutiae matching

Mathematical definition of the problem

Minutiae sets:

$$\mathcal{T} = \{\mathbf{m}_1, \mathbf{m}_2, \ldots, \mathbf{m}_M\}, \quad \mathbf{m}_i = \{x_i, y_i, \theta_i\}, \quad i = 1, \ldots, M$$
$$\mathcal{P} = \{\mathbf{m}'_1, \mathbf{m}'_2, \ldots, \mathbf{m}'_N\}, \quad \mathbf{m}'_j = \{x'_j, y'_j, \theta'_j\}, \quad j = 1, \ldots, N$$

Two minutiae match if:

$$s(\mathbf{m}'_j, \mathbf{m}_i) = \sqrt{(x'_j - x_i)^2 + (y'_j - y_i)^2} \leqslant r_0$$
$$d(\mathbf{m}'_j, \mathbf{m}_i) = \min\left(|\theta'_j - \theta_i|, 360^\circ - |\theta'_j - \theta_i|\right) \leqslant \theta_0$$

Minutiae matching

Mathematical definition of the problem

Finding a mapping function:

$$f : \mathbf{m}'_j \mapsto \mathbf{m}''_j$$

$$\mathbf{m}'_j = \{x'_j, y'_j, \theta'_j\}, \quad \mathbf{m}''_j = \{x''_j, y''_j, \theta'_j + \theta\}, \quad j = 1, \dots, N$$

typically only shift and rotation are used, that is:

$$\begin{bmatrix} x''_j \\ y''_j \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x'_j \\ y'_j \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

Minutiae matching

Mathematical definition of the problem

Aim function:

$$\max_{\Delta x, \Delta y, \theta, p} \sum_{i=1}^{N} h\Big(f_{\Delta x, \Delta y, \theta}(\mathbf{m}'_{p(i)}), \mathbf{m}_i\Big)$$

where

$$h(\mathbf{m}''_j, \mathbf{m}_i) = \begin{cases} 1 & \text{if } s(\mathbf{m}''_j, \mathbf{m}_i) \leqslant r_0 \text{ and } d(\mathbf{m}''_j, \mathbf{m}_i) \leqslant \theta_0 \\ 0 & \text{otherwise} \end{cases}$$

and p defines the minutiae correspondence in the maps $\mathcal{T}$ i $\mathcal{P}$.

Minutiae matching

Mathematical definition of the problem

Remarks on p function:

Minutiae matching

Mathematical definition of the problem

Remarks on p function:

- $p(i) = j$ does not mean that two minutiae "match" in h sense

Minutiae matching

Mathematical definition of the problem

Remarks on p function:

- $p(i) = j$ does not mean that two minutiae "match" in h sense
- $p(i) = j$ when $\mathbf{m}_i \in \mathcal{T}$ has a correspondent minutia $\mathbf{m}'_j \in \mathcal{P}$

Minutiae matching

Mathematical definition of the problem

Remarks on p function:

- $p(i) = j$ does not mean that two minutiae "match" in h sense
- $p(i) = j$ when $\mathbf{m}_i \in \mathcal{T}$ has a correspondent minutia $\mathbf{m}'_j \in \mathcal{P}$
- when $p(i) \neq j$ for $i = 1, \ldots, M$ then minutia $\mathbf{m}'_j \in \mathcal{P}$ has no correspondent minutiae in $\mathcal{T}$

Minutiae matching

Mathematical definition of the problem

Remarks on p function:

- $p(i) = j$ does not mean that two minutiae "match" in h sense
- $p(i) = j$ when $\mathbf{m}_i \in \mathcal{T}$ has a correspondent minutia $\mathbf{m}'_j \in \mathcal{P}$
- when $p(i) \neq j$ for $i = 1, \ldots, M$ then minutia $\mathbf{m}'_j \in \mathcal{P}$ has no correspondent minutiae in $\mathcal{T}$
- p is a bijection: each minutia $\mathbf{m}'_j \in \mathcal{P}$ has a single correspondent minutia $\mathbf{m}_i \in \mathcal{T}$ (and minutiae that have no correspondence are skipped)

Minutiae matching

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- p is a bijection: each minutia $\mathbf{m}'_j \in \mathcal{P}$ has a single correspondent minutia $\mathbf{m}_i \in \mathcal{T}$ (and minutiae that have no correspondence are skipped)
- in practice p is unknown, and brute-force solutions (checking every minutiae pair) yield exponential calculation times

Compensation of linear deformations

Example approach: use of Hough transform

1. Popular and fast
2. Converts point-pattern matching problem into detecting peaks in the so called **Hough space** or **parameter space**

   in fingerprint matching: location, rotation and scale of the minutiae maps
3. Independent 'experts' vote for their best combination of discretized parameter values

   each 'expert' increases elements in the discretized parameter space, the so called **accumulator space**, depending on its 'expertise'
4. Maximum element of the accumulator space suggests the winning combination of the discretized parameters

Compensation of linear deformations

Example approach: use of Hough transform

Assumptions:

Compensation of linear deformations

Example approach: use of Hough transform

Assumptions:

- transformation defined by the quadruple $(\Delta x, \Delta y, \theta, s)$: shift, rotation and scale

Compensation of linear deformations

Example approach: use of Hough transform

Assumptions:

- transformation defined by the quadruple $(\Delta x, \Delta y, \theta, s)$: shift, rotation and scale
- we use discretized values of $\Delta x, \Delta y, \theta, s$, namely: $\Delta x^+, \Delta y^+, \theta^+$ oraz s^+

Compensation of linear deformations

Example approach: use of Hough transform

Incrementation of the accumulator space elements:

Compensation of linear deformations

Example approach: use of Hough transform

Incrementation of the accumulator space elements:

$\mathbf{A}[\Delta x^+, \Delta y^+, \theta^+, s^+] = 0$ for all $\Delta x^+, \Delta y^+, \theta^+$ and s^+

Compensation of linear deformations

Example approach: use of Hough transform

Incrementation of the accumulator space elements:

$\mathbf{A}[\Delta x^+, \Delta y^+, \theta^+, s^+] = 0$ for all $\Delta x^+, \Delta y^+, \theta^+$ and s^+

for every $\mathbf{m}_i$, for every $\mathbf{m}'_j$, for every θ^+

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$\mathbf{A}[\Delta x^+, \Delta y^+, \theta^+, s^+] = 0$ for all $\Delta x^+, \Delta y^+, \theta^+$ and s^+

for every $\mathbf{m}_i$, for every $\mathbf{m}'_j$, for every θ^+

if $d(\theta'_j + \theta^+, \theta_i) \leqslant \theta_0$

Compensation of linear deformations

Example approach: use of Hough transform

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$\mathbf{A}[\Delta x^+, \Delta y^+, \theta^+, s^+] = 0$ for all $\Delta x^+, \Delta y^+, \theta^+$ and s^+

for every $\mathbf{m}_i$, for every $\mathbf{m}'_j$, for every θ^+

if $d(\theta'_j + \theta^+, \theta_i) \leqslant \theta_0$

for every s^+

$$\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \end{bmatrix} - s^+ \begin{bmatrix} \cos\theta^+ & -\sin\theta^+ \\ \sin\theta^+ & \cos\theta^+ \end{bmatrix} \begin{bmatrix} x'_j \\ y'_j \end{bmatrix}$$

Compensation of linear deformations

Example approach: use of Hough transform

Incrementation of the accumulator space elements:

$\mathbf{A}[\Delta x^+, \Delta y^+, \theta^+, s^+] = 0$ for all $\Delta x^+, \Delta y^+, \theta^+$ and s^+

for every $\mathbf{m}_i$, for every $\mathbf{m}'_j$, for every θ^+

if $d(\theta'_j + \theta^+, \theta_i) \leqslant \theta_0$

for every s^+

$$\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \end{bmatrix} - s^+ \begin{bmatrix} \cos\theta^+ & -\sin\theta^+ \\ \sin\theta^+ & \cos\theta^+ \end{bmatrix} \begin{bmatrix} x'_j \\ y'_j \end{bmatrix}$$

$\Delta x^+ =$ quantization(Δx), $\Delta y^+ =$ quantization(Δy)

$\mathbf{A}[\Delta x^+, \Delta y^+, \theta^+, s^+] = \mathbf{A}[\Delta x^+, \Delta y^+, \theta^+, s^+] + 1$

Compensation of linear deformations

Example approach: use of Hough transform

Winning combination of the discretized parameters:

$$(\Delta x^*, \Delta y^*, \theta^*, s^*) = \underset{\Delta x^+, \Delta y^+, \theta^+, s^+}{\operatorname{argmax}} \mathbf{A}(\Delta x^+, \Delta y^+, \theta^+, s^+)$$

Compensation of linear deformations

Example approach: use of Hough transform

Winning combination of the discretized parameters:

$$(\Delta x^*, \Delta y^*, \theta^*, s^*) = \underset{\Delta x^+, \Delta y^+, \theta^+, s^+}{\operatorname{argmax}} \mathbf{A}(\Delta x^+, \Delta y^+, \theta^+, s^+)$$

Useful modifications:

1. Simultaneous, weighted incrementation of $\mathbf{A}(\Delta x^+, \Delta y^+, \theta^+, s^+)$ and its neighbors
2. Use of parallel computing (parallelization of Hough transform is very easy)
3. Coarse-to-fine approaches

End of week remainders

1. Solve the quiz #2
   - deadline: Monday, Sept. 15, 11:00 AM
2. Attend the practical class on Monday, Sept. 15
   - bring an USB Flash or external hard drive to the class, it will be required to copy your biometric data and MATLAB programs (the package should not exceed 20 MB)
   - your biometric data will not be posted to any webpage due to security reasons